Investor Sentiment and the Mean-variance Relation

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Abstract

This paper examines the effects of investor sentiment on the mean-variance relation. Behind the well-known ambiguous relation between the mean and variance, the relation is significantly positive in the low sentiment periods and close-to-zero in the high sentiment periods. The results are robust for four volatility models. The empirical findings are consistent with economic intuitions. In the low sentiment periods, the price is damped down and the mean of return is pushed up. The market price is a compromise between rational traders and irrational traders. Hence the stocks are undervalued but irrational traders still believe that the stocks are overvalued. Rational investors invest more in stocks and irrational ones invest less. The ratio of the mean to the variance must be higher, which attracts more wealth of rational investors. Vice verse, the relation is lower in the high sentiment periods. We propose a general equilibrium model to formalize these intuitions. Furthermore, our empirical results show that the market's reaction to volatility is not homogenous through time but depends on investor sentiment. The whole market cares more about volatility when sentiment is low and less when sentiment is high. Besides the evidence from the mean-variance relation, the finding is confirmed by the empirical results of the return-innovation relation. The relation between the returns and the volatility innovations are much more negative in the low sentiment periods.

1 Introduction

Extensive studies have been done about the relation between the mean and the variance of stock market returns under the influence of Merton's (1973) ICAPM. Merton suggests that the conditional excess returns of the stock market should be positively correlated with the market's conditional variance:

$$E_t(R_{t+1}) = a + bVar_t(R_{t+1})$$

where b is the coefficient of the relative risk aversion of the representative agent.[1] The economic intuition behind this positive correlation is straightforward. The risk averse investors dislike volatility. High volatility reduces the current price and hence increases the future return. Investors receive compensation for bearing volatility.

But no conclusive evidence is found in the stock market data for this theoretic and intuitive relation. With different volatility models, divergent conclusions are reached. French, Schwert and Stambaugh (1987), Baillie and DeGennaro (1990), Campbell and Hentschel (1992) and Ghysels, Santa-Clara and Valkanov (2005) find an positive relation between the conditional mean and the conditional variance. Campbell (1987), Nelson (1991), Whitelaw (1994), Lettau and Ludvigson (2003) and Brandt and Kang (2004) find a negative relation. Turner, Starts and Nelson (1989), Glosten, Jagannathan and Runkle (1993), Harvey (2001) and MacKinlay and Park (2004) find both a positive and a negative relation.

The ambiguous empirical evidence leaves an embarrassment to finance theory. Although risk aversion and the dislike of the volatility are viewed as facts in most theories, there is no clear empirical evidence in the stock market data.

In classic finance theory, investor sentiment plays no role. Economists feel safe to ignore irrational investors based on the following two arguments. First, the irrational investors are met by rational investors, who trade against them. The process drives the stock price close to its fundamental value. However, De Long et al (1990) argue

[1] Abel (1988), Backus and Gregory (1993), and Gennotte and Marsh (1993) construct models where a negative expectation-variance relation is consistent with the equilibrium.

that arbitrageurs are likely to be risk averse and to have reasonably short horizons. The authors set up an overlapping generation model with arbitrageurs who hold correct beliefs, and noise traders who hold wrong beliefs. They reach the conclusions that arbitrage is limited and the stock price can be away from its fundamental value. Second, even if irrational investors may drive the price away from the fundamental value temporarily, they run out of wealth in the long run since they lose their money to rational investors. However, new noise traders could enter the market in the real world. Hence, although individual noise traders may die out sooner or later, the population could survive and remain relative stable. Moreover, Yan (2005) claims that although noise traders run out of wealth in the equilibrium without the entry of new ones, the process takes a very long time. His simulation results show that it takes several hundred years for noise traders with very wrong beliefs to lose half of their wealth. Baker and Wurgler (2005) construct an investor sentiment index from a set variables measuring the effects of noise trader sentiment on stock market. The forty-year index shows no decreasing trend in the effects of noise traders.

With the recognition of the existence of investor sentiment and the limits of arbitrage, investor sentiment should affect the mean-variance relation. In the stock market, risk-averse investors decide their holdings of stock based on how their volatility bearing is compensated. They would invest more when the ratio of the mean to variance is higher. When irrational traders are pessimistic (low sentiment periods), the price is damped down and the mean of return is pushed up. The market price is a compromise between rational traders and irrational traders. Hence the stocks are undervalued but irrational traders still believe that the stocks are overvalued. Rational investors invest more in stocks and irrational ones invest less. The ratio of the mean to the variance must be higher, which attracts more wealth of rational investors. Vice verse, when irrational investors are optimistic (high sentiment periods), the ratio of the mean to the variance should be lower.

A general equilibrium model with heterogenous beliefs is constructed to formalize the above intuitions. There are two types of investors in the market: arbitrageurs and noise traders. Both types are risk averse. The only difference is that noise traders hold incorrect beliefs about the fundamental value. In the equilibrium, when noise traders happen to have correct beliefs (zero sentiment), the Sharpe ratio is a positive constant. This is consistent with Merton's ICAPM. The model also shows that the Sharpe ratio is decreasing function of investor sentiment. Hence, the model implies that the mean-variance relation should be positive in the low sentiment periods. Moreover, the relation in the high sentiment periods should be lower than that of the low sentiment periods. However, whether the relation in the high sentiment periods is still positive, zero or even negative depends on the magnitude of sentiment. Hence, although all the investors are risk averse and dislike volatility, the divergence of beliefs can drive the mean-variance relation dramatically different. The investors receive more compensation for bearing volatility in the low sentiment periods than in the high sentiment periods. Hence the whole market exhibits more dislike of volatility in the low sentiment periods and less in the high sentiment periods.

Theoretical predictions are strongly supported by the empirical results. With the investor sentiment index proposed by Baker and Wurgler (2005), we analyze the mean-variance relation in the low and high sentiment periods. There is a significantly positive mean-variance relation in the low sentiment periods, and a close-to-zero relation in the high sentiment periods. The results are robust for four popular volatility models. We also analyze the relation between the returns and the innovations of volatility. The return-innovation relation results provide further evidence for the market's reaction to volatility. If the market cares more about the volatility in the low sentiment periods, the price should react more strongly to unexpected changes of volatility. A volatility shock should more strongly reduce the current price. The returns should have a more negative relation with contemporaneous innovations of volatility in the low sentiment periods. The empirical results support that the market dislikes the volatility more in the low sentiment periods than in the high sentiment periods. There is a significantly negative relation between the returns and the volatility innovations in the low sentiment periods. The relation is significantly less negative in the high sentiment periods. The results are also robust across the four volatility models.

So far as we know, this is the first paper to examine the effects of investor sentiment on the mean-variance relation. We find that there exists a hidden pattern behind the well-known ambiguous relation between the mean and variance. The relation is significantly positive in the low sentiment periods and close to zero in the high sentiment periods. The empirical results are consistent with economic intuitions. In the low sentiment periods, the stocks are undervalued and rational investors would invest more in the stocks. The mean-variance relation must be higher to attract more wealth of rational investors. Vice verse, the relation is lower in the high sentiment periods. We build a general equilibrium model to formalize the above intuitions. Our theory model further shows that investor sentiment is the source to drive the mean-variance relation to vary through time, which is strongly supported by the empirical results.

Furthermore, our empirical results show that the market's reaction to volatility is not homogenous through time but depends on investor sentiment. The whole market cares more about volatility when sentiment is low and less when sentiment is high. Besides the evidence from the mean-variance relation, the finding is confirmed by the empirical results of the return-innovation relation. The relation between the returns and the volatility innovations are much more negative in the low sentiment periods.

The rest of the paper is organized as follows: Section 2 presents our theory model and its implications. Section 3 introduces the investor sentiment index. Section 4 describes the methodology of the empirical approach. Section 5 reports the empirical results. Section 6 presents the conclusions.

2 The Model

In this section, we present a dynamic general equilibrium model with investor sentiment. The theory is minimized for the need of understanding the economic intuitions and empirical results. It deviates from the standard Lucas tree model by including a noise trader, who has incorrect beliefs about the fundamental value of stock. Our theoretical model shares some similarities with existing models of heterogeneous beliefs. For example, Detemple and Murthy (1994) develop a continuous time production economy in which traders have heterogeneous beliefs about parameters. Basak (2000) builds a model based on Detemple and Murthy (1994) and analyzes equilibrium effects of extraneous risk. Basak (2004) provides an excellent review of asset pricing models with heterogeneous beliefs. Our model setup is most related to Yan (2005). Yan (2005) constructs a general equilibrium model with constant misperception of the noise trader. He focuses his analysis on the survival of the irrational investor. In this paper, we analyze the economy with a more general sentiment process and focus on the impact of investor sentiment on the mean-variance relationship.

2.1 Information Structure

We consider a continuous-time pure-exchange economy with an infinite horizon. The uncertainty is represented by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathcal{P})$ on which a one-dimensional Brownian motion $B(t)$ is defined, where $\{\mathcal{F}_t\}$ denotes the augmented filtration generated by the Brownian motion $B(t)$. Assume there exists a stock which is a claim to the following exogeneous aggregrate dividend, or consumption, process $D(t)$.

$$\frac{dD(t)}{D(t)} = \mu_D dt + \sigma_D dB(t)$$

where μ_D and σ_D are positive constant. Assume there are two investors $(i = 1, 2)$ who commonly observe this dividend process. However, they have different beliefs about its underlying structure. Investor 1, the arbitrageur, holds the correct probability beliefs $\mathcal{P}^1 \equiv \mathcal{P}$. Investor 2, the noise trader, holds the wrong probability beliefs $\mathcal{P}^2$ which is equivalent with $\mathcal{P}$. By Girsanov theorem, there exist a unique process $\delta(t) \in \mathcal{L}^2$ such that $B_2(t) \equiv B(t) - \int_0^t \delta(s)\, ds$ is a Brownian motion under $\mathcal{P}^2$.

Hence, from the arbitrageur's point of view, the dividend has the following dynamics

$$\begin{aligned}\frac{dD(t)}{D(t)} &= \mu_{1,D}(t)\,dt + \sigma_D dB_1(t) \\ \text{where } \mu_{1,D} &\equiv \mu_D, \qquad B_1(t) \equiv B(t)\end{aligned}$$

However, from the noise trader's point of view, the dynamics of the dividend is

$$\begin{aligned}\frac{dD(t)}{D(t)} &= \mu_{2,D}(t)\,dt + \sigma_D dB_2(t) \\ \text{where } \mu_{2,D} &= \delta(t)\,\sigma_D + \mu_{1,D}, \quad \delta(t) = \frac{\mu_{2,D} - \mu_{1,D}}{\sigma_D}\end{aligned}$$

When $\delta(t)$ is positive, the noise trader is optimistic since he overestimates the expected growth rate of the dividend. On the other hand, when $\delta(t)$ is negative, he is pessimistic. In the model, $\delta(t)$ is defined as the noise trader's sentiment at time t.

2.2 Consumption Space and Securities Markets

There is a single perishable good in the economy. The consumption space $\mathcal{C}$ is given by the set of non-negative consumption rate processes c with $\int_0^T c(t)\,dt < \infty$ for any $T > 0$. The investment opportunities are represented by two long-lived securities, one risky stock and one locally riskless bond. The bond price process $B(t)$ is given by

$$dB(t) = B(t)\,r(t)\,dt, \text{ with } B(0) = 1$$

The stock price has the following dynamics

$$\begin{aligned}\frac{dS(t) + D(t)\,dt}{S(t)} &= \mu_1(t)\,dt + \sigma(t)\,dB(t) \\ &= \mu_2(t)\,dt + \sigma(t)\,dB_2(t)\end{aligned}$$

where the first equation is the price dynamics perceived by the arbitrageur and the second equation is the price dynamics perceived by the noise trader. The coefficients $\{r(t), \mu(t) \equiv \mu_1(t), \mu_2(t), \sigma(t)\}$ are determined endogenously in equilibrium. Although two investors have different beliefs about the expected stock return, they agree on the stock price path

$$\mu(t)\,dt + \sigma(t)\,dB(t) = \mu_2(t)\,dt + \sigma(t)\,dB_2(t)$$

Hence, we have

$$\frac{\mu_2(t) - \mu(t)}{\sigma(t)} = \delta(t)$$

Therefore, the misperception on dividend growth transfers to the misperception on the Sharpe ratio of the stock.

2.3 Trading Strategies

Trading takes place continuously and there are no market frictions. Investors' trading strategies (α_i, θ_i) satisfy

$$\int_0^T |\alpha_i(t) B(t) r(t)| \, dt + \int_0^T |\theta_i(t) S(t) \sigma(t)|^2 \, dt + \int_0^T |\theta_i(t) S(t) \mu_i(t)|^2 \, dt < \infty$$

for any $T > 0$, where $\alpha_i(t)$ and $\theta_i(t)$ denote, respectively, the number of shares of the bond and the stock held by investor i. This is purely a technical condition.

2.4 Investors Preferences and Endowments

To keep our model simple, we assume that both investors have the same CRRA utility function

$$u_i(c(t)) = \frac{c(t)^{1-\gamma}}{1-\gamma} \text{ for } \gamma > 0$$

The investor i's objective is to maximize

$$\max_{c_i(t)} E_0^i \left[\int_0^\infty e^{-\rho t} u_i(c_i(t)) \, dt \right]$$

subject the budget constraint

$$dW_i(t) = W_i(t) r(t) \, dt - c_i(t) \, dt + \theta_i(t) S(t) [\mu_i(t) - r(t)] \, dt + \theta_i(t) S(t) \sigma(t) \, dB_i(t)$$

where $W_i(t) \geq 0$ is investor i's financial wealth at time t and ρ is the time preference parameter. Here, we use $E_t^i[\cdot]$ to deonte the conditional expectation operator relative to agent i's probability beliefs. At time 0, investor i is endowed with β_i shares of stock, $\beta_1 + \beta_2 = 1$, and $\beta_i \geq 0$.

2.5 Sentiment Process

We assume exogenously the sentiment process $\delta(t)$ follows a mean reverting dynamics[2]

$$d\delta(t) = -\alpha\delta(t)\,dt + \eta dB(t)$$

This process has both theoretical and empirical foundations. From the theoretical side, Scheinkman and Xiong (2003) argue that overconfidence would lead to a mean-reverting difference of opinions between different investors. Using their method, we could generate exactly the above sentiment process. From the empirical side, Baker and Wurgler (2005) build an investor sentiment index, which clearly follows a mean-reverting process.

2.6 Equilibrium and Implications

Definition: A competitive equilibrium is a price system $(B(t), S(t))$, consumption process $c_i(t)$, and portfolio $(\alpha_i(t), \theta_i(t))$ such that: (i) each investor optimizes his portfolio-consumption strategy. (ii) perceived security price processes are the same across investors. (iii) all markets clear.

We present an intuitive solution of the equilibrium in the paper and a rigorous proof for all the claims in the appendix. To characterize the competitive equilibrium, we adopt a technique first used by Cuoco and He (1994) and developed by Basak (2000). Basak (2000) demonstrates that in economies with heterogeneous beliefs, the equilibrium can be attained by constructing a representative investor with a stochastic weight process that captures the difference in investors' beliefs.

By the standard martingale approach, we can solve the equilibrium completely.

[2]We exclude any learning in the model for simplicity. Furthermore, a model with learning usually implies that investor sentiment converges to zero, which is inconsistent with the data. For simplicity, we also assume that the process is mean-reverting to zero. Our results do not depend on this assumption.

First, define the representative agent with the utility

$$\begin{aligned}U\left(c\left(t\right);\lambda\left(t\right)\right) &= \max_{c_1(t)+c_2(t)=c(t)} e^{-\rho t}u_1\left(c_1\left(t\right)\right)+\lambda\left(t\right)e^{-\rho t}u_2\left(c_2\left(t\right)\right) \\ &\equiv e^{-\rho t}u_1\left(c_1^*\left(t\right)\right)+\lambda\left(t\right)e^{-\rho t}u_2\left(c_2^*\left(t\right)\right)\end{aligned}$$

where $\lambda\left(t\right)>0$ captures the heterogeneity in beliefs and $c_i^*\left(t\right)$ is the optimal consumption for agent i. By calculus, we can show that

$$U_c\left(c\left(t\right);\lambda\left(t\right)\right)=\lambda\left(t\right)u_2'\left(c_2^*\left(t\right)\right)=u_1'\left(c_1^*\left(t\right)\right)$$

Furthermore, by the first order condition for agent i, the optimal consumption has to satisfy

$$e^{-\rho t}u_i'\left(c_i^*\left(t\right)\right)=\psi_i\xi_i\left(t\right)$$

where $\xi_i\left(t\right)$ is the pricing kernel perceived by agent i and ψ_i is some normalization constant which can be determined from budget condition. Hence,

$$\lambda\left(t\right)=\frac{u_1'\left(c_1^*\left(t\right)\right)}{u_2'\left(c_2^*\left(t\right)\right)}=\frac{\psi_1\xi_1\left(t\right)}{\psi_2\xi_2\left(t\right)}$$

$\lambda(t)$ is the ratio of investors' marginal utility at time t. Following Ito's lemma, we have

$$d\lambda\left(t\right)=\lambda\left(t\right)\delta\left(t\right)dB\left(t\right)$$

Hence, $\lambda\left(t\right)$ is a positive local martingale, and then a supermartingale. Under reasonable assumptions (e.g. mean-reverting) on the sentiment process $\delta(t)$, the weighting process $\lambda(t)$ is an honest martingale. In the homogenous case, $\xi_1\equiv\xi_2$, then $\lambda\left(t\right)$ is a constant.

For CRRA utility, we can calculate the representative agent's utility function,

$$\begin{aligned}U\left(c;\lambda\right) &= \max_{c_1+c_2=c}\left[\frac{c_1\left(t\right)^{1-\gamma}}{1-\gamma}+\lambda\frac{c_2\left(t\right)^{1-\gamma}}{1-\gamma}\right]\cdot e^{-\rho t} \\ &= \frac{c\left(t\right)^{1-\gamma}}{1-\gamma}\left(\frac{1}{1+\lambda^{\frac{1}{\gamma}}}\right)^{-\gamma}\cdot e^{-\rho t}\end{aligned}$$

The individuals' optimal consumptions are

$$c_1^*=\frac{\lambda^{-\frac{1}{\gamma}}}{1+\lambda^{-\frac{1}{\gamma}}}c\left(t\right);\qquad c_2^*=\frac{1}{1+\lambda^{-\frac{1}{\gamma}}}c\left(t\right).$$

Therefore, the pricing kernel of the arbitrageur is

$$
\begin{aligned}
\xi_1(t) &= \frac{1}{\psi_1} e^{-\rho t} u_1'(c_1(t)) \\
&= \frac{1}{\psi_1} e^{-\rho t} \left(\frac{1+\lambda(t)^{\frac{1}{\gamma}}}{D(t)} \right)^{\gamma}
\end{aligned}
$$

By Ito's lemma, we have

$$
\begin{aligned}
\frac{d\xi_1}{\xi_1} &= -\left(\rho + \gamma\mu_D + \gamma\sigma_D \delta(t) \frac{\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}} + \frac{\gamma-1}{2\gamma} \frac{\delta^2(t)\lambda^{\frac{1}{\gamma}}}{\left(1+\lambda^{\frac{1}{\gamma}}\right)^2} - \frac{\gamma(\gamma+1)}{2}\sigma_D^2 \right) dt \\
&\quad + \left(\frac{\lambda(t)^{\frac{1}{\gamma}}}{1+\lambda(t)^{\frac{1}{\gamma}}} \delta(t) - \gamma\sigma_D \right) dB(t)
\end{aligned}
$$

Since $\xi_1(t)$ is the pricing kernel and the arbitrageur has the correct beliefs, we have

$$
\frac{d\xi_1}{\xi_1} = -r(t)dt - \frac{\mu(t)-r(t)}{\sigma(t)} dB(t)
$$

By matching the diffusion coefficients of the above two equations, we have the following theorem.

Theorem 1 *For CRRA utility, we have the following relationship between the expected excess return and the volatility.*

$$
\mu(t) - r(t) = \left[\gamma\sigma_D - \frac{\lambda(t)^{\frac{1}{\gamma}}\delta(t)}{1+\lambda(t)^{\frac{1}{\gamma}}} \right] \cdot \sigma(t)
$$

Proof: See Appendix.

γ is the coefficient of the relative risk aversion, σ_D is the diffusion coefficient of the dividend process, $\delta(t)$ is investor sentiment, and $\lambda(t)$ is the stochastic weighting process in the representative agent utility function. If the arbitrageur holds most of the wealth ($\lambda(t) \to 0$) or the noise trader happens to have correct beliefs ($\delta(t) = 0$), the Sharpe ratio should be a positive constant, $\gamma\sigma_D$. This conclusion is consistent with ICAPM. If everyone in the market holds correct beliefs, volatility is compensated with a positive constant price. Another clear message delivered by Theorem 1 is that the Sharpe ratio is decreasing function of investor sentiment. As the noise trader becomes more optimistic, the investors receive less compensation for volatility. In the extreme case, the Sharpe ratio can be negative when the noise trader is very optimistic. Theorem 1 shows that the Sharpe ratio is a nonlinear function of investor sentiment and the stochastic weighting process, $\lambda(t)$. $\lambda(t)$ can be viewed as a proxy for the ratio of the noise trader's wealth to the arbitrageur's wealth. Hence the relation depends on both noise trader sentiment and participation. Theorem 1 is very difficult to test directly if it is not impossible. However, it supplies two testable predictions, which we examine in the sections that follow. First, in the low sentiment periods (negative sentiment), the mean-variance relation should be positive. Second, the relation in the high sentiment periods should be lower than the low sentiment periods. It is worthy of note that the proof of Theorem 1 doesn't require any assumptions on the sentiment process $\delta(t)$. Hence the theorem is valid under very general settings.

In our economy, the noise trader can not survive, since $\lambda(t)$ converges to zero almost surely. However, Yan (2005) shows that, even with constant misperception of noise traders, it takes on average four hundred years for noise traders to lose half of his wealth. Under our settings, noise trader sentiment is a mean-reverting process, which makes noise traders easier to survive. Hence, it should not matter much for our analysis.

3 Investor Sentiment Index

In this paper, we use the investor sentiment index proposed by Baker and Wurgler (2005) to identify the low and high sentiment periods. Baker and Wurgler (2005) form a composite index with six measures of investor sentiment. Since every measure of sentiment contains some sentiment information and non-sentiment-related information, they use the first principal component to capture the common variation in the six measures.

The six measures are the closed-end fund discount, the NYSE share turnover, the number and average first-day returns on IPOs, the equity share in new issues and the dividend premium. The closed-end fund discount is the average difference between the net asset value and the market price. Lee, Shleifer and Thaler (1991) show the existence of the closed-end fund puzzle and argue that investor sentiment is the culprit. NYSE turnover is computed from the ratio of share volume to average share number. Baker and Stein (2004) propose a model with short-sales constrains and overconfidence investors. The model implies turnover comoves with investor sentiment. The activities of the IPO market are often viewed as related to investor sentiment. Stigler (1964) and Ritter (1991) argue that the return of the IPO firms can be explained by market timing. Cornelli, Goldreich and Ljungqvist (2004) suggest that sentiment demand has significant impact on the price of IPO firms. Baker and Wurgler (2000) document that the share of equity issues in total equity and debt issues has strong predictive ability for stock returns, which is consistent with the market timing hypothesis. Hence it should be influenced by investor sentiment. The dividend premium is the log difference of the average market-to-book ratios of dividend payers and nonpayers. Baker and Wurgler (2004a and 2004b) propose a catering theory of dividends: investor sentiment drives the prices of dividend payers or nonpayers away from the fundamental values, which provides rational managers the incentive to cater. Since the dividend premium is the measure of uniformed demand for dividend-paying shares driven by sentiment, it should capture the information of investor sentiment.

To remove the business-cycle component from the sentiment index, Baker and Wurgler (2005) regress each of the raw sentiment measures on the industrial production index growth, consumer durables growth, nondurables growth and service growth, employment growth and a dummy variable for NBER recessions. The orthogonalized measures, which are the residuals from the above regressions, should be cleaner measures for investor sentiment. Finally, the composite sentiment index is standardized to have zero mean and unit variance.

In this paper, we use the updated investor sentiment index, which begins in 1962 and ends in 2003.[3] The solid line in Figure 1 is the composite sentiment index. The

[3]Baker and Wurgler (2005) also construct the index with the six raw measures. They show that

index captures most anecdotal accounts of fluctuations in sentiment.[4] After the 1961 crash of growth stocks, investor sentiment is low, rising to a subsequent peak in the 1968 and 1969 electronic bubble. By the mid-1970s, sentiment is again low. In the late 1970s, sentiment level picks up and reaches a peak in the biotech bubble. Sentiment drops in the late 1980s and goes up again in the early 1990s reaching a peak in the internet bubble.

Table 1 reports the correlations of the sentiment measures. The results show that there is a strong common component in the six sentiment measures. In *Panel A*, we find most of correlations of the raw measures are significant. The correlations between the index and the six raw measures are high and significant at a 5% level. In *Panel B*, with controlling macroeconomics conditions, the correlations are even stronger. All the correlations among the orthoganalized measures are significant at a 10% level and almost two thirds of them are significant at a 1% level. The correlation with the index are very high and all significant at a 1% level.

A criticism to Baker and Wurgler's index is that there may be alternative hypotheses for most of the six sentiment measures. The index could identify some other economic source instead of investor sentiment. However, Table 1 shows that there is a common source behind all the individual sentiment measures. Although it is not difficult to provide a hypothesis that other economic variable is behind some single sentiment measure, it is far more challenging to propose a unified theory to argue that something other than investor sentiment is the common source of all six measures. Unless such a unified alternative can be reasonably formulated, investor sentiment seems to be the most plausible component identified by Baker and Wurgler's index.

Moreover, there is evidence that the empirical results based on Baker and Wurgler's sentiment index are consistent with behavior hypotheses. In Baker and Wurgler (2005), they analyze the effects of investor sentiment on the cross-sectional pattern of stocks with their own index. They find that the cross-sectional patterns vary with

the two indexes have little difference, which also has very minor influence on our results.

[4]For the detailed discussions about the anecdotal history of investor sentiment, see Baker and Wurgler (2005).

investor sentiment in a way behavioral hypothesis predicts. Yuan (2005) uses Baker and Wurgler's index to examine the predictive ability of sentiment for stock returns. The high value of the sentiment index predicts low future returns and the low value predicts high future returns, which is consistent with the behavior theory. With Baker and Wurgler's index, this paper also find significant empirical results that are consistent with the economic intuitions and our theoretical model. Hence, our paper provides some additional evidence that Baker and Wurgler's index indeed identifies investor sentiment other than any other economic source.

4 Methodology

Four volatility models are used to analyze two relations: the mean-variance relation[5] and the relation between the returns and the innovations of volatility. The four volatility models are the rolling window model, the Mixed Data Sampling approach (MIDAS), GARCH(1,1) and asymmetric GARCH(1,1). The purpose of our paper is not to evaluate and compare different volatility models. Instead, all the models are treated equally to analyze the mean-variance relation. Before going through the details of the four models, we first introduce two common variables used in all the four models.

The first variable is a dummy variable for the high sentiment periods. In this paper, we examine the relation between monthly returns and monthly volatility. However, the sentiment index is annual data. We use beginning-of-period sentiment as a proxy for investor sentiment.[6] So the sentiment value at the beginning of a year is assigned to every month in that year. Then we define the months with the top

[5]Our model implies the relation between the mean and standard deviation. However, almost all the existing literature focus the analysis on the mean-variance relation. We examine the mean-variance relation in this paper. The relation between the mean and standard deviation has the same empirical patterns as the mean-variance relation.

[6]One advantage is that the dummy variable is in the last month information set. So only ex ante information is added when the dummy variables are incorporated to models.

half sentiment values as the high sentiment periods, and the other half as the low sentiment periods.[7] D_{t+1} is defined as the dummy variable for the high sentiment periods, i.e. D_{t+1} is 1 if the month $t+1$ is in the high sentiment periods. D_{t+1} is in the information set of the month t, since it is derived from the beginning-of-period sentiment value.

The second variable is the monthly realized variance. Following French, Schwert and Stambaugh (1987), we estimate the monthly realized variance with the within-month daily returns:

$$\sigma_t^2 = \frac{22}{N_t} \sum_{d=1}^{N_t} r_{t-d}^2 \ ,$$

where r_{t-d} is the demeaned daily return in the month, the corresponding subscript $t-d$ is for the date t minus d days, N_t is the number of trading days in the month t and 22 is the approximate number of trading days in one month. The above estimator is justified theoretically by Anderson et al (2003), who show that it is an unbiased and efficient estimator of actual volatility.

The rest of this section is as follows: Subsection 4.1 discusses the mean-variance relation and the return-innovation relation. Subsection 4.2, 4.3 and 4.4 present the specifications of the conditional variance and the innovation of volatility for the four models.

4.1 Two Relations

The previous literature analyzes the mean-variance relation in the equation:

$$R_{t+1} = a + bVar_t(R_{t+1}) + \epsilon_{t+1} \ ,$$

where R_{t+1} is the monthly excess return and $Var_t(R_{t+1})$ is the conditional variance. To test whether investor sentiment has significant effects on the mean-variance relation, the following equation is analyzed:

$$R_{t+1} = a_1 + b_1 Var_t(R_{t+1}) + a_2 D_{t+1} + b_2 D_{t+1} Var_t(R_{t+1}) + \epsilon_{t+1} \ ,$$

[7] With the mean of sentiment as the cutting point, we obtain the same results.

where D_{t+1} is the dummy variable for the high sentiment periods. So a_1 and b_1 are the intercept and the coefficient in the low sentiment periods. a_2 and b_2 are the differences of the intercepts and the coefficients between the high sentiment periods and the low sentiment periods respectively.

To analyze the relation between the returns and the volatility innovations, the following equation is examined:

$$\begin{aligned} R_{t+1} &= a_1 + b_1 Var_t(R_{t+1}) + c_1 Var(R_{t+1})^u \\ &\quad + a_2 D_{t+1} + b_2 D_{t+1} Var_t(R_{t+1}) + c_2 D_{t+1} Var(R_{t+1})^u + \epsilon_{t+1} \ , \end{aligned}$$

where $Var_t(R_{t+1})$ is the conditional variance and $Var(R_{t+1})^u$ is the contemporaneous volatility innovation.

We have two optional measures for volatility innovations. One is the unexpected change of current volatility. The second is the unexpected change of future volatility. Since the volatility process is highly persistent, the two measures should be highly correlated. Each of the measures has its own advantage and disadvantage. The second one is more plausible theoretically because what investors really care about is future volatility. But in some volatility models, additional assumptions are needed to get the volatility exceeding the next period. Hence there is some risk of misspecifications. In this paper, we choose the volatility innovation measure with the following strategy. If no additional assumption is needed to estimate the volatility exceeding the next period, we use the unexpected change of future volatility as the proxy. Otherwise, the unexpected change of current volatility is selected.

4.2 Rolling Window Model

A natural method to estimate the conditional variance is the rolling window model (for example, French, Schwert and Stambaugh (1987), MacKinlay and Park (2004) and Ghysels, Santa-Clara and Valkanov (2005)). The rolling window model uses the realized variance of this month as the conditional variance for the next-month return.

So the conditional variance is

$$Var_t(R_{t+1}) = \sigma_t^2 = \frac{22}{N_t} \sum_{d=1}^{N_t} r_{t-d}^2 \ .$$

French, Schwert and Stambaugh (1987) argue that the difference between realized variance and conditional variance is a good proxy for volatility innovation. They call the difference "unpredictable component of volatility". In the rolling window model, we use the unpredictable component of the volatility as the proxy for volatility innovation:

$$Var(R_{t+1})^u = \sigma_{t+1}^2 - Var_t(R_{t+1}) = \sigma_{t+1}^2 - \sigma_t^2 \ .$$

The unpredictable component of volatility is the unexpected change of current volatility. With the additional assumption that the volatility follows a random walk process, this measure here is also the unexpected change of future volatility.

4.3 Mixed Data Sampling Approach

The rolling window model models the conditional variance as the sum of approximately 22 squared daily demeaned returns with equal weights. Ghysels, Santa-Clara and Valkanov (2005) argue that, although the rolling window model is straightforward, a better estimator can be achieved with longer horizon daily returns and optimal weights. They construct a new estimator of conditional variance with the Mixed Data Sampling approach (MIDAS). The MIDAS estimator of the conditional variance is as follows:

$$Var_t(R_{t+1}) = 22 \sum_{d=0}^{250} w_d r_{t-d}^2 \ ,$$

where

$$w_d(\kappa_1, \kappa_2) = \frac{exp\{\kappa_1 d + \kappa_2 d^2\}}{\sum_{i=0}^{250} exp\{\kappa_1 i + \kappa_2 i^2\}} \ ,$$

r_{t-d} is the demeaned daily return[8] and the corresponding subscript $t-d$ is for the date t minus d days. The daily data of the previous 250 days, approximately one year, is used to model the monthly conditional variance. w_d is the weight on r_{t-d}^2. κ_1 and κ_2 are the parameters controlling the weights. The weight function has several good properties. First, the weight is positive. Second, the sum of the weight is always equal to one. Ghysels, Santa-Clara and Valkanov (2005) further argue that the weight function can provide flexible shape. The parameters in MIDAS can be estimated with the maximum likelihood method.

To examine the return-innovation relation, we also use the unpredictable component of volatility as a proxy of volatility innovation. With the maximum likelihood estimators of κ_1 and κ_2, the conditional variance can be estimated. Then the difference between the realized variance and the conditional variance are calculated as volatility innovation.

4.4 GARCH and Asymmetric GARCH

Bollerslev (1986) proposes the GARCH model based on the ARCH model developed by Engle (1982). In recent years, the GARCH model is extensively used in modeling the volatility of the stock market returns. Nelson (1991) and Glosten, Jaganathan and Runkle (1993) argue that the GARCH model should have more flexibility, in which positive shocks and negative shocks can have different influence on volatility. To solve the this problem, Glosten, Jaganathan and Runkle (1993) propose asymmetric GARCH. GARCH(1,1) and asymmetric GARCH(1,1) are the third and fourth volatility models in this paper. In GARCH(1,1), the conditional variance is modeled as follows:

$$Var_t(R_{t+1}) = \omega + \alpha\epsilon_t^2 + \beta Var_{t-1}(R_t) \ ,$$

[8] To be consistent with the realized variance estimator and the rolling window model, daily demeaned return is used here. The daily demeaned return is computed by subtracting the within-month mean return from the daily raw return. Ghysels, Santa-Clara and Valkanov (2005) use daily raw return instead. This modification has very minor effects on our empirical results.

where $Var_t(R_{t+1})$ is the conditional variance and ϵ_t is the residual in the mean-variance equation in Subsection 4.1. In asymmetric GARCH(1,1), the conditional variance is modeled as follows:

$$Var_t(R_{t+1}) = \omega + \alpha_1 \epsilon_t^2 + \alpha_2 I_t \epsilon_t^2 + \beta Var_{t-1}(R_t) \ ,$$

where I_t is the dummy variable for positive shocks, i.e. I_t is 1 when ϵ_t is positive.

To check the return-innovation relation, we need to estimate volatility innovations. In GARCH(1,1) and asymmetric GARCH(1,1), we can calculate the future variance exceeding the next period. Without any new assumptions, we can compute the unexpected changes of future variance, which is used as the proxy for volatility innovation here. Daily data is used to improve the volatility estimation. The details are as follows. We first fit the daily returns with the simple GARCH(1,1):

$$r_{t+1}^{raw} = \mu + \epsilon_{daily,t+1} \ ,$$

$$h_{t+1} = \omega + \alpha \epsilon_{daily,t}^2 + \beta h_t \ ,$$

where r_{t+1}^{raw} is the daily raw return and h_{t+1} is the conditional variance of the daily returns. With the estimations from daily GARCH(1,1), the monthly conditional variance and the monthly volatility innovation are calculated:

$$Var_t(R_{t+1}) = E_t(\sum_{d=1}^{22} h_{t+d}) \ ,$$

$$Var(R_{t+1})^u = Var_{t+1}(R_{t+2}) - Var_t(R_{t+2}) = E_{t+1}(\sum_{d=1}^{22} h_{t+1+d}) - E_t(\sum_{d=23}^{44} h_{t+d}) \ ,$$

where R_t is the monthly excess return, h_t is the conditional variance of the daily returns and the corresponding subscript $t+d$ is for the date t plus d days. Then the above estimations are used to analyze the return-innovation equation in Subsection 4.1. For asymmetric GARCH(1,1), the procedures are the same. The difference is that the daily conditional variance is modeled as asymmetric GARCH(1,1).

5 Empirical Results

5.1 Data and Summary Statistics

The annual investor sentiment index starts in 1962 and ends in 2003. The investor sentiment data is provided by Malcolm Baker and Jeffrey Wurgler. We use the NYSE-AMEX equal-weighted index as a proxy for the stock market and the return of one month T-bill as a proxy for the interest rate. The monthly equal-weighted index returns, the daily equal-weighted index returns and the monthly T-bill returns are obtained from CRSP for the period from January 1963 to December 2004.

Table 2 displays summary statistics of the monthly excess returns and the realized variance. We report the summary statistics in the whole sample, in the low sentiment periods and in the high sentiment periods.

Panel A shows that the moments of the returns in the low sentiment periods are quite different from those in the high sentiment periods . In the whole sample, the mean of the excess return is 0.773% and the variance is 0.294×10^{-2}. The excess returns are negatively skewed and leptokurtic. The mean of the excess returns is 1.396% in the low sentiment periods and 0.150% in the high sentiment periods. The returns are much higher in the low sentiment periods, which is consistent with economic intuitions and our model. The variance of the returns in the low sentiment periods, 0.250×10^{-2}, is slightly lower than the variance in the high sentiment periods, 0.333×10^{-2}. *Panel B* reports the moments of the monthly realized variance. The low sentiment periods are clearly less volatile than the high sentiment periods. All the moments of the realized variance in the low sentiment periods are much smaller than those in the high sentiment periods.

The skewness and kurtosis of stock returns exhibit interesting patterns. The skewness in the whole sample is -0.067. The skewness is 0.958 in the low sentiment periods and -0.654 in the high sentiment periods. The overall negative-skewness property seems mainly to result from the high sentiment periods. The kurtosis is 7.479 for the whole sample, 10.547 for the low sentiment periods and 5.149 for the high sentiment periods. The low sentiment periods have a fatter tail. Hence the well-documented negative skewness and fat tail properties of the stock returns can be viewed in a new way: the negative skewness in the high sentiment periods and the fat tail property in the low sentiment periods.

In fact, our theoretical model implies that the skewness of the returns is positive in the low sentiment periods and negative in the high sentiment periods. Under the assumption that sentiment follows a mean-reverting process, the distribution of sentiment, conditional on the low sentiment periods, should have a longer tail on the left side. Hence the sentiment is left skewed in the low sentiment periods. A lower sentiment implies a higher expected return given other quantities fixed in our theoretical model. This is also intuitive since lower sentiment pushes down the stock price and increases the future excess return. Therefore, the returns in the low sentiment periods should be right skewed. The skewness of the return in the low sentiment periods should be positive. Vice versa, the skewness in the high sentiment periods should be negative.

5.2 Mean-variance Relation

Table 3 and Table 4 report the estimates and t statistics for the mean-variance relation. The results from the rolling window model and MIDAS are in Table 3. The results from GARCH(1,1) and asymmetric GARCH(1,1) are in Table 4.

Our model predicts that the mean-variance relation should be positive in the low sentiment periods and that the relation should be lower in the high sentiment periods. Our empirical results provide strong supports for the theoretical predictions. In the rolling window model, b, the coefficient of the conditional variance in the model without the sentiment effects, is -0.299. The t statistic is -0.33. There is an ambiguous mean-variance relation. When sentiment effects are considered, b_1 is 13.075 and b_2 is -13.714. The t statistic is 1.94 and -2.02 respectively. b_1 is significant for the one-sided test and b_2 is significant for the two-sided test. $b_1 + b_2$, the coefficient in the high sentiment periods, is -0.639 [9]. The t statistic is -1.06. So there is a significantly positive relation in the low sentiment periods and an insignificantly negative relation in the high sentiment periods.

The positive relation in the low sentiment periods is not only statistically significant but also economically significant. The standard deviation of the realized variance is 7.779×10^{-4} in the low sentiment periods. One standard deviation increase in variance is associated with an increase of the monthly returns by 1.017%.

We get similar results in MIDAS. In the model without the sentiment effects, b is 3.232. The t statistic is 1.47, which is not significant. b_1 is 19.861 and b_2 is -18.147. The t-statistics are 4.55 and -3.53 respectively. Both are significant. $b_1 + b_2$ is 1.714. The t statistic is 0.633. The relation is significantly positive in the low sentiment periods and significantly lower in the high sentiment periods. Moreover, the relation in the high sentiment periods are not statistically different from zero. κ_1 and κ_2 are respectively -0.018 and 0.048×10^{-3}. The estimated weight system is different from the equal-weighted system.[10]

In Table 4, similar results are also found from GARCH(1,1) and asymmetric GARCH(1,1). In GARCH(1,1), b is 4.461. The t statistic is 1.95. b_1 is 7.566 and b_2 is -3.531. The t statistics are 2.28 and -0.86 respectively. $b_1 + b_2$ is 4.038 with the t statistic, 1.18. In asymmetric GARCH(1,1), b is 4.080. The t statistic is 1.84. In the model with the sentiment dummy variables, b_1 is 11.453 and b_2 is -8.029. The t statistics are 2.59 and -1.76. $b_1 + b_2$ is 3.424 with the t statistic, 1.18.

With the updated data, our results confirm some findings by the existing literature. First, the mean-variance relation is ambiguous if sentiment effects are not considered. Second, the estimations are sensitive to the selection of volatility model. However, in all four volatility models, we reach the same conclusion: there is a significantly positive relation in the low sentiment periods and close-to-zero relation in the high sentiment periods. The empirical results are consistent with our theory model.

[9] $b_1 + b_2$, the coefficient in the high sentiment periods is not reported in the tables. Instead, we report b_1, the coefficient in low sentiment periods, and b_2, the difference of the coefficients.

[10] When κ_1 and κ_2 are both zeros, the estimated weight system is equal-weighted.

Investors receive more compensation for bearing volatility in the low sentiment periods. Moreover, our new findings also provide insight for the market's reaction to volatility. The whole market exhibits more dislike of volatility when sentiment is low.

Although the estimates from the four volatility models seem to be different, they have very close economic implications. In the four volatility models, the standard deviation of estimated variance process in the low sentiment periods is 7.778×10^{-4}, 4.780×10^{-4}, 15.503×10^{-4} and 10.564×10^{-4} respectively. The relation is 13.075, 19.861, 7.556 and 11.453. Hence one standard deviation increase in variance is associated with an increase of the monthly returns by 1.017%, 0.949%, 1.173% and 1.210% in the four models.

We also examine the mean-variance relation for the value-weighted index. The monthly and daily NYSE-AMEX value-weighted index returns are obtained from CRSP. The results from the four volatility models are reported in Table 5. All the estimations of b_1 are positive and all the estimations of b_2 are negative. Most of them are significant. The results here verify our findings.

5.3 Return-innovation Relation

The empirical results from the return-innovation relation confirm that the market dislikes volatility more in the low sentiment periods. The relation between the return and contemporaneous volatility innovations is more negative in the low sentiment periods. Table 6 and Table 7 report the estimates and t statistics for the return-innovation relation. The results from the rolling window model and MIDAS are reported in Table 6. The results from GARCH(1,1) and asymmetric GARCH(1,1) are present in Table 7.

In the rolling window model, c_1, the coefficient of the volatility innovation in the low sentiment periods, is -22.625. The t statistic is -3.10. c_2, the difference between the coefficients in the high sentiment periods and the coefficient in the low sentiment periods, is 15.543. The t statistic is 2.09. Hence there is a significantly negative relation in the low sentiment periods and a significantly less negative relation in the high sentiment periods. It seems that the unexpected change of volatility have effects on the current price both in the low and high sentiment periods. However, the market reacts much stronger in the low sentiment periods. It dislikes volatility more when sentiment is low.

In MIDAS, we get similar empirical results. c_1 is -24.674. The t statistic is -3.54. c_2 is 17.456. The t statistic is 2.46. The coefficient in the low sentiment periods is significantly negative and the difference is significantly positive. In GARCH(1,1) and asymmetric GARCH(1,1), similar results are also found. c_1 is -21.656 in GARCH(1,1) and -26.198 in asymmetric GARCH(1,1). The t statistic is -4.14 and -8.47 respectively. c_2 is 12.760 in GARCH(1,1) and 15.395 in asymmetric GARCH(1,1). The t statistic is 2.31 and 3.95 respectively. The estimates and t statistics for the daily models are showed in *Panel B* in Table 7.

Again, the same empirical conclusions are reached in the four volatility models. There is a significantly negative relation between the return and the contemporaneous volatility innovations in the low sentiment periods. The relation is significantly less negative in the high sentiment periods. Furthermore, with the four different volatility models and the two different volatility innovation measures, we get very close estimations. c_1 ranges from -21 to -27 and c_2 ranges from 12 to 18. The empirical results are very robust across the different volatility models and the different volatility innovation measures.

The empirical results of the return-innovation relation provide additional evidence for the market's reaction to volatility. The whole market dislikes volatility more and reacts more strongly to volatility in the low sentiment periods. A volatility shock reduces the current price more in the low sentiment periods than in the high sentiment periods.

The results for the value-weighted index are reported in Table 8. All the estimations of c_1 and c_2 have the correct sign. Moreover, all of them are significant. The results of the value-weighted index strongly support our findings about the return-innovation relation.

5.4 Mean-variance Relation as a Linear Function of Sentiment

Theorem 1 shows that the Sharpe ratio is a complicated nonlinear function of investor sentiment as follows:

$$\mu(t) - r(t) = \left[\gamma\sigma_D - \frac{\lambda(t)^{\frac{1}{\gamma}}\delta(t)}{1+\lambda(t)^{\frac{1}{\gamma}}}\right]\cdot\sigma(t)$$

where $\lambda(t)$ is the stochastic weighting function in the representative agent utility function and $\delta(t)$ is investor sentiment. In general, the relation is nonlinear. However, in some special cases, it is a linear function of investor sentiment.

If the noise trader holds most of the wealth in the market, $\lambda(t) \to \infty$. Then the above relation is as follows.

$$\mu(t) - r(t) = [\gamma\sigma_D - \delta(t)]\cdot\sigma(t)$$

Since the composite sentiment index is a linear transformation of the real sentiment process, the relation is a linear function of the sentiment index. Under this setting, the representative agent has correct beliefs on average. However he may be optimistic or pessimistic sometimes.

The relation is also a linear function of sentiment if $\lambda(t)$ is constant over time. $\lambda(t)$ can be viewed as a proxy for the ratio of noise trader's wealth to arbitrageur's wealth. If the participation of noise traders does not vary across time, the linear relation should approximately hold.

In this subsection, we examine the results in the specification where the mean-variance relation is a linear function of the sentiment index. The test is not only a robust check for our main empirical results. It also provides another empirically feasible way to test whether the mean-variance relation depends on investor sentiment. The caveat is that the theory model does not imply such a linear function in general. It is valid only in some special cases.

The empirical model is as follows:

$$R_{t+1} = a_0 + a_1\delta_t + (b_0 + b_1\delta_t)Var_t(R_{t+1}) + \epsilon_{t+1}\ ,$$

where δ_t is the sentiment value at the end of last year and $Var_t(R_{t+1})$ is the conditional variance of the monthly excess returns. The conditional variance is estimated by the four volatility models. In GARCH(1,1) and asymmetric GARCH(1,1), both monthly and daily data are used to estimate the conditional variance. b_1 is expected to be negative, which implies that the mean-variance relation is a decreasing function of sentiment. b_0 is expected to be positive. There should be a positive relation when sentiment is zero.

Table 9 reports the estimations and t statistics. All the estimations for b_0 and b_1 have the correct sign and most of them are significant. All the estimations of b_1 are negative. Three out of the six are significant for the two-sided test. One is significant for the one-sided test. All the estimations of b_0 are positive. Two of them are significant for the two-sided test. One is significant for the one-sided test. Furthermore, the estimations are also economically significant. Since the sentiment index has unit variance, b_1 is the magnitude of the mean-variance relation change associated with one standard deviation change in sentiment. b_1 ranges from -3.883 to -1.154. Sentiment clearly has significant effects on the mean-variance relation. The results here strongly support our main empirical finding that investor sentiment affects the mean-variance relation.

6 Conclusion

This paper analyzes the effects of investor sentiment on the mean-variance relation. We build a general equilibrium model that implies that the mean-variance relation should depend on investor sentiment. The relation should be positive in the low sentiment periods and lower in the high sentiment periods. Hence, in the low sentiment periods, investors receive more compensation for bearing volatility and the whole market exhibits more dislike of volatility.

The mean-variance relation analysis have in the past been very sensitive to the choice of volatility models. When considering investor sentiment, we find a hidden pattern of the relation that is robust across the volatility models. The mean-variance relation is significantly positive in the low sentiment periods and close to zero in the high sentiment periods. The empirical results are consistent with our theoretical predictions.

Moreover, the empirical results show that the market reaction to volatility is not homogeneous across time. The market reacts more strongly to volatility in the low sentiment periods. The mean-variance relation results imply that volatility more strongly affect the price in the low sentiment periods than in the high sentiment periods. The empirical results of the return-innovation relation confirm the finding. The relation is significantly negative in the low sentiment periods and less negative in the high sentiment periods.

Appendix

A. Proof of Theorem 1

The proof is a variation of Karatzas, Ioannis, John Lehoczky, and Steven Shreve (1990) with heterogeneous agents. In the following, we provide a detailed proof. Assume that $\{r(t), \mu(t), \sigma(t), c_i(t), \alpha_i(t), \theta_i(t)\}$ is an equilibrium, then agent i's optimization problem is

$$\max_{c_i(t)} E^i \int_0^\infty e^{-\rho t} \frac{c_i(t)^{1-\gamma}}{1-\gamma} dt$$

subject to

$$dW_i(t) = W_i(t) r(t) dt - c_i(t) dt + \theta_i(t) S(t) [\mu_i(t) - r(t)] dt + \theta_i(t) S(t) \sigma(t) dB_i(t)$$

Since the market is complete, by the martingale approach (see Cox and Huang (1989) or Karatzas, Ioannis, John Lehoczky, and Steven Shreve (1987)), it is equivalent for agent i to solve the following static problem

$$\max_{c_i(t)} E^i \int_0^\infty e^{-\rho t} \frac{c_i(t)^{1-\gamma}}{1-\gamma} dt$$

subject to

$$E^i \int_0^\infty \xi_i(t) c_i(t) dt \leq \beta_i E^i \int_0^\infty \xi_i(t) D(t) dt,$$

where $\xi_i(t)$ ($\xi_i(0) = 1$) is the pricing kernel perceived by agent i with the following dynamics

$$\frac{d\xi_i(t)}{\xi_i(t)} = -r(t) dt - \frac{\mu_i - r(t)}{\sigma(t)} dB_i(t).$$

Therefore, by the first order condition,

$$e^{-\rho t} c_i^*(t)^{-\gamma} = \psi_i \xi_i(t) \tag{1}$$

where ψ_i can be determined from the following budget condition

$$E^i \int_0^\infty \xi_i(t) \left[\psi_i e^{-\rho t} \xi_i(t)\right]^{-\frac{1}{\gamma}} dt = \beta_i E^i \int_0^\infty \xi_i(t) D(t) dt$$

Let's define $\lambda(t)$ as

$$\lambda(t) = \frac{e^{-\rho t} c_1^*(t)^{-\gamma}}{e^{-\rho t} c_2^*(t)^{-\gamma}} = \frac{\psi_1 \xi_1(t)}{\psi_2 \xi_2(t)} \tag{2}$$

By Ito's lemma and the constraint $\frac{\mu_2(t)-\mu_1(t)}{\sigma(t)} = \delta(t)$, we have

$$
\begin{aligned}
d\lambda(t) &= d\left[\frac{\psi_1 \xi_1(t)}{\psi_2 \xi_2(t)}\right] \\
&= \lambda(t)\left[-r(t)\,dt - \frac{\mu_1(t)-r(t)}{\sigma(t)} dB_1(t)\right] \\
&\quad + \lambda(t)\left[r(t)\,dt + \left(\frac{\mu_2 - r(t)}{\sigma(t)}\right)^2 dt + \frac{\mu_2(t)-r(t)}{\sigma(t)} dB_2(t)\right] \\
&\quad - \lambda(t)\frac{\mu_1(t)-r(t)}{\sigma(t)}\frac{\mu_2(t)-r(t)}{\sigma(t)} dt \\
&= \lambda(t)\left[\delta(t)\frac{\mu_2(t)-r(t)}{\sigma(t)}\right] dt \\
&\quad + \lambda(t)\left[\frac{\mu_2(t)-r(t)}{\sigma(t)} dB_1(t) - \delta(t)\frac{\mu_2(t)-r(t)}{\sigma(t)} dt - \frac{\mu_1(t)-r(t)}{\sigma(t)} dB_1(t)\right] \\
&= \lambda(t)\,\delta(t)\,dB_1(t)
\end{aligned}
$$

From the market clearing condition, we have

$$
c_1^*(t) + c_2^*(t) = D(t) \tag{3}
$$

Then, combining equation (2) and (3), we have

$$
c_1^*(t) = \frac{D(t)}{1+\lambda(t)^{-\frac{1}{\gamma}}}; \qquad c_2^*(t) = \frac{\lambda(t)^{-\frac{1}{\gamma}} D(t)}{1+\lambda(t)^{-\frac{1}{\gamma}}} \tag{4}
$$

By equation (1) and (4),

$$
\begin{aligned}
\xi_1(t) &= \frac{1}{\psi_1} e^{-\rho t} c_1^*(t)^{-\gamma} \\
&= \frac{1}{\psi_1} e^{-\rho t}\left(\frac{1+\lambda(t)^{\frac{1}{\gamma}}}{D(t)}\right)^{\gamma}
\end{aligned}
$$

By Ito's lemma, we have

$$
\begin{aligned}
\frac{d\xi_1}{\xi_1} &= -\left(\rho + \gamma\mu_D + \gamma\sigma_D\delta(t)\frac{\lambda^{\frac{1}{\gamma}}}{1+\lambda^{\frac{1}{\gamma}}} + \frac{\gamma-1}{2\gamma}\frac{\delta^2(t)\lambda^{\frac{1}{\gamma}}}{\left(1+\lambda^{\frac{1}{\gamma}}\right)^2} - \frac{\gamma(\gamma+1)}{2}\sigma_D^2\right) dt \\
&\quad + \left(\frac{\lambda(t)^{\frac{1}{\gamma}}}{1+\lambda(t)^{\frac{1}{\gamma}}}\delta(t) - \gamma\sigma_D\right) dB(t)
\end{aligned}
$$

Since $\xi_1(t)$ is the pricing kernel and the arbitrageur has the correct beliefs, we have

$$\frac{d\xi_1}{\xi_1} = -r(t)dt - \frac{\mu(t) - r(t)}{\sigma(t)} dB(t)$$

By matching the diffusion coefficients of the above two equations, we obtain

$$\mu(t) - r(t) = \left[\gamma\sigma_D - \frac{\lambda(t)^{\frac{1}{\gamma}} \delta(t)}{1 + \lambda(t)^{\frac{1}{\gamma}}} \right] \cdot \sigma(t)$$

Therefore, we have completed the proof of Theorem 1.

B. The existence of the irrational beliefs that supports the mean-reverting sentiment process

In the following, we show that there exists a probability measure $\mathcal{P}^2$ which supports a mean-reverting investor sentiment process. Instead of checking the usual Novikov condition, we show that the sentiment process $\delta(t)$ satisfies the condition in Corollary 5.16 in Karatzas and Shreve (1991) (page 200). Indeed,

$$\begin{aligned}
\delta(t) &= \delta(0) e^{-\alpha t} + \eta \int_0^t e^{-\alpha(t-s)} dB(s) \\
&\leq |\delta(0)| + \eta \left(B(t) - \int_0^t B(s) \, de^{-\alpha(t-s)} \right) \\
&\leq |\delta(0)| + \eta \left(2 - e^{-\alpha t} \right) B^*(t) \\
&\leq (1 + B^*(t)) \cdot \max(\eta \left(2 - e^{-\alpha t} \right), |\delta(0)|)
\end{aligned}$$

By Corollary 5.16 in Karatzas and Shreve (1991), there exists a probability belief $\mathcal{P}^2$ which supports the sentiment process $\delta(t)$ since $Z(\delta(t))$ is a martingale.

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Figure 1: The Investor Sentiment Index from 1962 to 2003

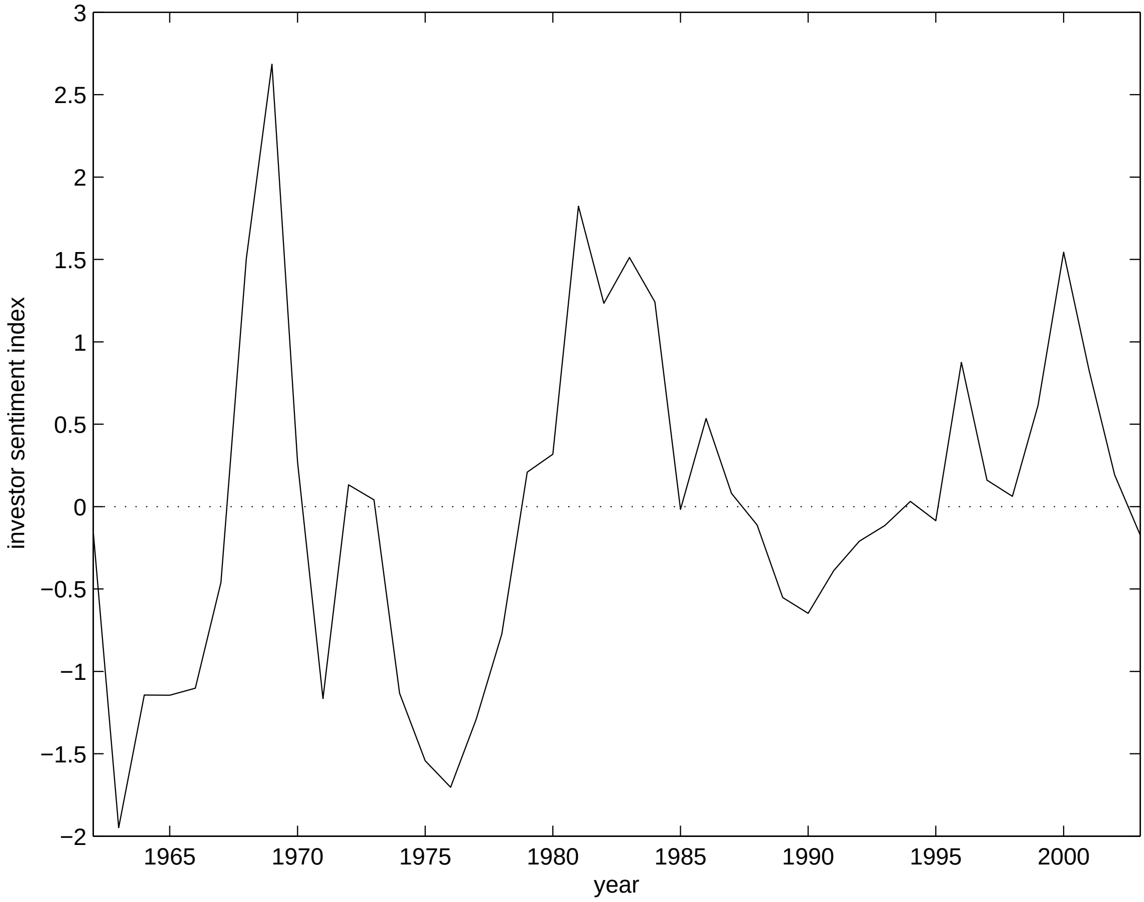

Table 1: The Correlations Among Six Sentiment Measures

	Panel A Raw Measures						
	$CEFD$	$TURN$	$NIPO$	$RIPO$	S	P^{D-ND}	*Sentiment*
$CEFD$	1.000						-0.602^a
$TURN$	-0.305^b	1.000					0.682^a
$NIPO$	-0.511^a	0.310^b	1.000				0.602^a
$RIPO$	-0.386^b	0.474^a	0.266^c	1.000			0.786^a
S	0.018	0.218	0.244	0.174	1.000		0.371^b
P^{D-ND}	0.520^a	-0.499^a	-0.506^a	-0.591^a	-0.083	1.000	-0.770^a
	Panel B Controlling Macro Conditions						
	$CEFD^{\perp}$	$TURN^{\perp}$	$NIPO^{\perp}$	$RIPO^{\perp}$	$S^{\perp}$	$P^{D-ND\perp}$	*Sentiment*
$CEFD^{\perp}$	1.000						-0.622^a
$TURN^{\perp}$	-0.339^b	1.000					0.652^a
$NIPO^{\perp}$	-0.422^a	0.267^c	1.000				0.668^a
$RIPO^{\perp}$	-0.434^a	0.516^a	0.422^a	1.000			0.804^a
$S^{\perp}$	-0.270^c	0.363^b	0.484^a	0.400^a	1.000		0.552^a
$P^{D-ND\perp}$	0.329^b	-0.513^a	-0.470^a	-0.659^a	-0.269^c	1.000	-0.755^a

$CEFD$ is the year-end, value-weighted average discount on close-end funds. $TURN$ is detrended natural log turnover. Turnover is the ratio of share volume to average shares from the NYSE Fact Book. $NIPO$ is the annual number of IPO firms. $RIPO$ is the annual first-day returns of IPO firms. S is the annual equity issuance divided by annual equity plus debt issuance. P^{D-ND} is the year-end log ratio of the value-weighted average market-to-book ratios of dividend payers and nonpayers. In *Panel B*, to control macro conditions, we regress the six sentiment measures on the growth of industry production, the growth of durable, nondurable and services consumption, the growth of employment and a dummy variable for NBER recessions. The orthogonalized measures, labeled with a $\perp$, are the residuals from the regressions. *Sentiment* is the first principal component of the six orthogonalized measures. The sample period is from 1962 to 2003. a, b and c denote statistical significance at 1%, 5% and 10% level.

Table 2: Summary Statistics of Monthly Excess Returns and Realized Variance

Panel A Monthly Excess Returns

	Mean $\times 10^2$	Variance $\times 10^2$	Skewness	Kurtosis
Whole sample	0.773	0.294	-0.067	7.479
Low sentiment	1.396	0.250	0.958	10.547
High sentiment	0.150	0.333	-0.654	5.149

Panel B Monthly Realized Variance

	Mean $\times 10^3$	Variance $\times 10^6$	Skewness	Kurtosis
Whole sample	1.069	5.832	15.013	282.091
Low sentiment	0.689	0.565	2.793	12.551
High sentiment	1.449	10.833	11.535	158.755

The monthly excess returns are computed from the monthly NYSE-AMEX equal-weighted index returns and one month T-bill returns. The realized variance is computed from the with-in month demeaned daily NYSE-AMEX equal-weighted index returns.The sample period is from January 1963 to December 2004.

Table 3: The Monthly Excess Returns Against the Conditional Variance From the Rolling Window Model and MIDAS

	a (a_1)	b (b_1)	a_2	b_2		
	Panel A	Rolling Window Model				
Without sentiment	0.008	-0.299				
(5), (7)	(3.10)	(-0.33)				
With sentiment	0.005	13.075	-0.002	-13.714		
(6), (7)	(1.15)	(1.94)	(-0.38)	(-2.02)		
	a (a_1)	b (b_1)	a_2	b_2	κ_1	κ_2 $\times 10^3$
		Panel B	MIDAS			
Without sentiment	0.004	3.232			-0.018	0.048
(5), (8)	(2.23)	(1.47)			(-2.36)	(1.66)
With sentiment	-0.001	19.861	-0.000	-18.147	-0.020	0.053
(6), (8)	(-0.21)	(4.55)	(-0.09)	(-3.53)	(-2.77)	(1.90)

$$R_{t+1} = a + bVar_t(R_{t+1}) + \epsilon_{t+1} \tag{5}$$

$$R_{t+1} = a_1 + b_1 Var_t(R_{t+1}) + a_2 D_{t+1} + b_2 D_{t+1} Var_t R_{t+1} + \epsilon_{t+1} \tag{6}$$

$$Var_t(R_{t+1}) = 22 \sum_{d=1}^{N_t} \frac{1}{N_t} r_{t-d}^2 \tag{7}$$

$$Var_t(R_{t+1}) = 22 \sum_{d=0}^{250} w_d r_{t-d}^2 \quad w_d(\kappa_1, \kappa_2) = \frac{exp\{\kappa_1 d + \kappa_2 d^2\}}{\sum_{i=0}^{250} exp\{\kappa_1 i + \kappa_2 i^2\}} \tag{8}$$

R_{t+1} is the monthly excess returns on the NYSE-AMEX equal-weighted index. r_{t-d} is the daily demeaned NYSE-AMEX equal-weighted index returns (the daily returns minus the within-month mean). N_t is the number of trading days in the month t. D_{t+1} is the dummy variable for the high sentiment periods. When the sentiment is high, it is 1. The sample period is from January 1963 to December 2004. The numbers in the parenthesis are t-statistics from the Newey-West standard error estimators with 12 lags in *Panel A* and t-statistics from the MLE standard error estimators in *Panel B*.

Table 4: The Monthly Excess Returns Against the Conditional Variance From GARCH(1,1) and Asymmetric GARCH(1,1)

	a (a_1)	b (b_1)	a_2	b_2	ω $\times 10^3$	α	β	
Panel A GARCH(1,1)								
Without sentiment	-0.003	4.461			0.150	0.086	0.868	
(9), (11)	(-0.51)	(1.95)			(2.13)	(3.06)	(22.93)	
With sentiment	-0.006	7.566	-0.001	-3.531	0.167	0.087	0.859	
(10), (11)	(-0.74)	(2.28)	(-0.06)	(-0.86)	(2.19)	(3.14)	(22.13)	
	a (a_1)	b (b_1)	a_2	b_2	ω $\times 10^3$	α_1	α_2	β
Panel B Asymmetric GARCH(1,1)								
Without sentiment	-0.003	4.080			0.241	0.126	-0.097	0.838
(9), (12)	(-0.45)	(1.84)			(2.45)	(2.86)	(-1.86)	(18.39)
With sentiment	-0.017	11.453	0.011	-8.029	0.266	0.124	-0.103	0.831
(10), (12)	(-1.46)	(2.59)	(0.88)	(-1.76)	(3.29)	(3.16)	(-2.63)	(21.84)

$$R_{t+1} = a + bVar_t(R_{t+1}) + \epsilon_{t+1} \tag{9}$$

$$R_{t+1} = a_1 + b_1Var_t(R_{t+1}) + a_2D_{t+1} + b_2D_{t+1}Var_t(R_{t+1}) + \epsilon_{t+1} \tag{10}$$

$$Var_t(R_{t+1}) = \omega + \alpha\epsilon_t^2 + \beta Var_{t-1}(R_t) \tag{11}$$

$$Var_t(R_{t+1}) = \omega + \alpha_1\epsilon_t^2 + \alpha_2I_t\epsilon_t^2 + \beta Var_{t-1}(R_t) \tag{12}$$

R_{t+1} is the monthly excess returns on the NYSE-AMEX equal-weighted index. $Var_t(R_{t+1})$ is the conditional variance. D_{t+1} is the dummy variable of the investor sentiment. When the sentiment is high, it is 1. I_t is the dummy variable of positive shocks. The sample period is from January 1963 to December 2004. The numbers in the parenthesis are t-statistics.

Table 5: The Mean-variance Relation for the Value-weighted Index

	a_1	b_1	a_2	b_2
Rolling window	-0.000	8.650	0.003	-9.361
	(-0.00)	(2.22)	(0.72)	(-2.38)
MIDAS	-0.002	10.413	0.002	-9.593
	(-0.66)	(2.62)	(0.46)	(-2.14)
GARCH(1,1)	-0.007	9.696	-0.005	-2.080
	(-0.68)	(1.50)	(-0.43)	(-0.29)
Asy GARCH(1,1)	-0.019	15.703	0.028	-17.109
	(-1.54)	(2.02)	(2.34)	(-2.41)

$$R_{t+1} = a_1 + b_1 Var_t(R_{t+1}) + a_2 D_{t+1} + b_2 D_{t+1} Var_t R_{t+1} + \epsilon_{t+1}$$

R_{t+1} is the monthly excess returns on the NYSE-AMEX value-weighted index. $Var_t(R_{t+1})$ is the conditional variance. D_{t+1} is the dummy variable for the high sentiment periods. When the sentiment is high, it is 1. The sample period is from January 1963 to December 2004. The numbers in the parenthesis are t-statistics from the Newey-West standard error estimators with 12 lags or from the MLE standard error estimators.

Table 6: The Monthly Excess Returns Against the Conditional Variance and the Unpredictable Component of Variance From the Rolling Window Model and MIDAS Model

	a_1	b_1	c_1	a_2	b_2	c_2
Rolling window	0.014	-1.395	-22.625	0.002	-5.664	15.543
(13), (14)	(2.11)	(-0.12)	(-3.10)	(0.35)	(-0.49)	(2.09)
MIDAS	0.001	16.885	-24.674	0.005	-19.280	17.456
(13), (15)	(0.06)	(1.54)	(-3.54)	(0.53)	(-1.66)	(2.46)

$$
\begin{aligned}
R_{t+1} &= a_1 + b_1 Var_t(R_{t+1}) + c_1 Var(R_{t+1})^u \\
&\quad + a_2 D_{t+1} + b_2 D_{t+1} Var_t R_{t+1} + c_2 D_{t+1} Var(R_{t+1})^u + \epsilon_{t+1} \qquad (13)
\end{aligned}
$$

$$
Var_t(R_{t+1}) = 22 \sum_{d=1}^{N_t} \frac{1}{N_t} r_{t-d}^2 \qquad (14)
$$

$$
Var_t(R_{t+1}) = 22 \sum_{d=0}^{250} w_d r_{t-d}^2 \quad w_d(\kappa_1, \kappa_2) = \frac{exp\{\kappa_1 d + \kappa_2 d^2\}}{\sum_{i=0}^{250} exp\{\kappa_1 i + \kappa_2 i^2\}} \qquad (15)
$$

R_{t+1} is the monthly excess returns on the NYSE-AMEX equal-weighted index. $Var_t(R_{t+1})$ is the conditional variance. $Var(R_{t+1})^u$ is the unpredictable component of the variance (the realized variance minus the conditional variance). κ_1 and κ_2 are estimated from the MIDAS model without the sentiment dummy variable in Table 2 ($\kappa_1 = -0.018$ and $\kappa_2 = 0.048 \times 10^{-3}$). r_{t-d} is the daily demeaned NYSE-AMEX equal-weighted index returns (the daily returns minus the within-month mean). N_t is the number of trading days in the month t. D_{t+1} is the dummy variable for the high sentiment periods. When the sentiment is high, it is 1. The sample period is from January 1963 to December 2004. The numbers in the parenthesis are t-statistics from the Newey-West standard error estimators with 12 lags.

Table 7: The Monthly Excess Returns Against the Conditional Variance and the Innovations of Future Volatility From GARCH(1,1) and Asymmetric GARCH(1,1)

Panel A						
	a_1	b_1	c_1	a_2	b_2	c_2
GARCH	0.001	6.277	-21.656	0.010	-11.633	12.760
(16), (17), (18)	(0.07)	(0.57)	(-4.14)	(0.99)	(-1.05)	(2.31)
Asymmetric GARCH	0.009	-0.008	-26.198	0.002	-5.089	15.395
(16), (17), (19)	(1.36)	(-0.00)	(-8.47)	(0.25)	(-0.66)	(3.95)

Panel B Daily Estimations					
	a $\times 10^3$	ω $\times 10^6$	$\alpha(\alpha_1)$	α_2	β
GARCH	1.090	1.802	0.181		0.794
(17), (18)	(19.56)	(9.49)	(17.25)		(68.93)
Asymmetric GARCH	1.009	1.869	0.231	-0.123	0.797
(17), (19)	(18.25)	(9.73)	(16.17)	(-10.68)	(67.61)

$$R_{t+1} = a_1 + b_1 Var_t(R_{t+1}) + c_1 Var(R_{t+1})^u$$
$$+ a_2 D_{t+1} + b_2 D_{t+1} Var_t R_{t+1} + c_2 D_{t+1} Var(R_{t+1})^u + \epsilon_{t+1} \quad (16)$$
$$r^{raw}_{t+1} = \mu + \epsilon_{daily,t+1} \quad (17)$$
$$h_{t+1} = \omega + \alpha \epsilon^2_{daily,t} + \beta h_t \quad (18)$$
$$h_{t+1} = \omega + \alpha_1 \epsilon^2_{daily,t} + \alpha_2 I_t \epsilon^2_{daily,t} + \beta h_t \quad (19)$$

R_{t+1} is the monthly excess returns on the NYSE-AMEX equal-weighted index. $Var_t(R_{t+1})$ is the monthly conditional variance implied by daily GARCH(1,1) or asymmetric GARCH(1,1). $Var(R_{t+1})^u$ is the innovations of future volatility implied by daily GARCH(1,1) or asymmetric GARCH(1,1). r^{raw}_{t+1} is the daily raw return and h_{t+1} is the conditional variance of daily returns. D_{t+1} is the dummy variable for the high sentiment periods. I_t is the dummy variable for positive shocks. The sample period is from January 1963 to December 2004. The numbers in the parenthesis are t-statistics from the Newey-West standard error estimators with 12 lags in *Panel A* and t-statistics from the MLE standard error estimators in *Panel B*.

Table 8: The Return-innovation Relation for the Value-weighted Index

	a_1	b_1	c_1	a_2	b_2	c_2
Rolling window	0.009	-1.411	-15.679	0.002	-2.699	11.578
	(1.95)	(-0.24)	(-4.41)	(0.38)	(-0.47)	(3.28)
MIDAS	0.003	4.147	-21.050	0.005	-6.906	16.762
	(0.72)	(0.91)	(-6.08)	(0.86)	(-1.46)	(4.85)
GARCH	0.005	-2.051	-19.944	0.006	-1.931	14.748
	(0.89)	(-0.32)	(-3.71)	(0.86)	(-0.31)	(2.73)
Asy GARCH	0.010	-7.496	-30.901	0.000	3.901	25.637
	(2.10)	(-1.55)	(-8.59)	(0.06)	(0.81)	(6.70)

$$\begin{aligned} R_{t+1} &= a_1 + b_1 Var_t(R_{t+1}) + c_1 Var(R_{t+1})^u \\ &\quad + a_2 D_{t+1} + b_2 D_{t+1} Var_t R_{t+1} + c_2 D_{t+1} Var(R_{t+1})^u + \epsilon_{t+1} \end{aligned}$$

R_{t+1} is the monthly excess returns on the NYSE-AMEX value-weighted index. $Var_t(R_{t+1})$ is the conditional variance. $Var(R_{t+1})^u$ is the volatility innovation. D_{t+1} is the dummy variable for the high sentiment periods. When the sentiment is high, it is 1. The sample period is from January 1963 to December 2004. The numbers in the parenthesis are t-statistics from the Newey-West standard error estimators with 12 lags.

Table 9: Model the Mean-variance Relation as a Linear Function of Investor Sentiment

	a_0 $\times 10^2$	a_1 $\times 10^2$	b_0	b_1
Rolling window	0.661	-0.312	1.699	-2.748
	(2.49)	(-1.16)	(1.12)	(-2.20)
MIDAS	0.116	-0.323	6.777	-3.696
	(0.59)	(-1.67)	(2.95)	(-1.77)
monthly GARCH(1,1)	-0.443	-0.170	4.923	-1.154
	(-0.64)	(-0.29)	(1.95)	(-0.60)
monthly Asy GARCH(1,1)	-0.514	0.544	5.104	-3.883
	(-0.78)	(0.63)	(2.13)	(-1.18)
daily GARCH(1,1)	0.576	-0.199	2.022	-2.833
	(1.84)	(-0.70)	(1.03)	(-2.15)
daily Asy GARCH(1,1)	0.710	-0.247	0.974	-2.631
	(2.75)	(-0.88)	(0.75)	(-2.44)

$$R_{t+1} = a_0 + a_1\delta_t + (b_0 + b_1\delta_t)Var_t(R_{t+1}) + \epsilon_{t+1}$$

R_{t+1} is the monthly excess returns on the NYSE-AMEX equal-weighted index. δ_t is investor sentiment at the end of last year. $Var_t(R_{t+1})$ is the conditional variance. The sample period is from January 1963 to December 2004. The numbers in the parenthesis are t-statistics from the Newey-West standard error estimators with 12 lags or from the MLE standard error estimators.